Experimental Validation of State Estimation of Non-Linear Twin Rotor System using Extended Kalman Filter

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Abstract

In this paper, theoretical and experimental validation of state estimation capability of Extended Kalman Filter (EKF) is done on MIMO twin rotor system. Different immeasurable states are estimated. For theoretical validation, states with different behaviors (e.g. random, exponential, sinusoidal and abruptly changed) are generated and outputs are calculated. These outputs are invoked in EKF algorithm that provides an estimate of above generated states. Comparison between self-generated and estimated states is made. In practical, information of immeasurable process states is needed. Therefore experimental readings of inputs and outputs of the system are inserted in the EKF algorithm that provides the close estimate of desired immeasurable states. This experimental validation is carried out for open loop and closed loop twin rotor system data. The results show that EKF estimates are precise and fast convergent to the actual twin rotor states.

Keywords: Twin rotor system; state estimation; extended Kalman filter; nonlinear model; experimental validation

1. Introduction

In the most applications, internal states of a process are either immeasurable or inaccessible; rather, system outputs are measurable containing noise of certain magnitude. The true knowledge of internal states is necessary for system monitoring or their ultimate use in control design and fault diagnostics etc. In such situations, state estimators can be utilized to evaluate process states provided that the information of inputs and outputs is available. Kalman filter is one such a tool developed by R.E Kalman in 1960 [1]. Now a day, it is extensively used to estimate the immeasurable system states from noisy outputs. Noise corrupted outputs of the system act as an input to the filter. Thereafter, its algorithm recursively keeps adjusting its parameters to arrive at close estimate of the desired immeasurable states. Kalman Filter adjusts its parameters by averaging the noisy measurements [2]. It computes state estimates for linear systems. State can be stationary or nonstationary. Also system can be time invariant or varying. Moreover, its algorithm is rather intuitive, logical and easy to understand. Its stability analysis has been carried out in detail [3]. Also Kalman filter require comparatively less computational complexity in the galaxy of estimation algorithms [4]. Kalman filter has become a popular state and parameter estimation tool, due to its wide problem handling capabilities and large number of variants. It has been categorically used as an estimator, as a filter and as a data Fusion tool [5-6]. Major areas of Kalman applications include aerospace, data networks, navigations, digital and adaptive signal processing, fault diagnosis, parametric estimation, and dual estimation problems [5-6]. In aerospace, Kalman filter has been used for generation of nonlinear control law for gyroscope PWM torque loop, where it estimates rotor velocity and flux of an induction motor [7]. The EKF's performance is demonstrated using both a straight-and-level manoeuvre and a complicated manoeuvres recorded on-board a manoeuvrings F-15. In both cases, the state estimates of the EKF are similar to the results obtained from a coordinated flight model [8]. Many aerospace systems are characterized by nonlinear models as well as noisy and biased sensor measurements. Extended Kalman Filter (EKF) is a commonly used algorithm for recursive parameter identification due to its excellent filtering properties and is based on a first order approximation of the system dynamics. EKF has enabled aerodynamic specialists to estimate aerodynamic coefficients of projectiles. Estimation is based on three discrete flight measurements that include three spatial positions and three angular orientations. [12]. In direct vector control of induction machines, the

instantaneous rotor flux vector is measured using sensors, estimators or a combination of both. Since the basic Kalman filter is a state estimator, its use in vector-controlled schemes has received much attention, including reduced-order variants. [11]. Digital signal processing based EKF estimation has been done for speed and rotor position estimate of a PM synchronous motor [14]. In field oriented control induction motors, In order to know the instantaneous flux, accurate values of rotor resistance must be known. An Extended Kalman Filter (EKF) is employed for rotor resistance estimation [15].

Apart from aerospace applications, Kalman has also been extensively used. For example, the EKF used in radar tracking applications is computationally intensive leading to difficulties in broadband real-time applications. The EKF algorithm is analyzed in detail for reduction of number of computations [16]. Kalman filter has also been used for estimation of competing terminals in an IEEE 802.11 network export [17]. Fault Classification in products in production plants has been done via Kalman filter [19]. Extended Kalman filter is used for GPS C/A code tracking and interference rejection applications [20]. Kalman filter tuning capability is used to control the photo resist properties of a semiconductor while its manufacture, Kalman do its control job by precisely tuning itself to minimize the drift in process states values due to noise. This feature enables semiconductor manufacturer to design even micro scale integrated devices [21]. EKF has been used to estimate the linearized direct and indirect stiffness and damping force coefficients for bearings in rotor-dynamic applications from noisy measurements of the shaft displacement in response to imbalance and impact excitation [10].

The EKF can efficiently work in non-linear systems which makes it quite suitable especially for WSN applications [22]. Moreover, [23] has discussed the computational complexity of EKF, applications where requirement of analytical Jacobean and Hessian seem prohibitive. [23] Presented tools to facilitate an EKF2 with lower complexity as compared to the straightforward implementation using explicit Jacobean and Hessian [23].

One of the reason behind utilizing EKF is that the EKF is easier to implement and is faster than UKF in applications like Probabilistic Robotics etc. It brings saving in computational resources as these filters are implemented on embedded systems. The EKF being ubiquitous, is easy to tune and corresponding parameters are well understood. Finally, why bring a replacement? When EKF is already working in an existing environment. This is one of the reason why we used this variant of the filter [24]. Further studies on EKF can be seen in [25-26]. Finally, the studies related to dynamic system analysis related to renewable energy resources can be seen in [27-31].

The present paper uses EKF to estimate the states of twin rotor from the self-generated and experimental measurements of inputs and outputs. The paper is organized as follows. Section II describes the step by step development of standard and extended Kalman algorithms. The computational results for system model after applying EKF are discussed in Section III. Then the Simulation results of state estimation are presented for different self-generated state behaviors e.g. random, exponential, sinusoidal and abruptly changed are presented in section IV. In section V experimental validation results in open and closed loop are discussed.

2. Kalman filter algorithm

2.1 Development of Kalman filter algorithm

For estimation Kalman filter requires linear (linearized) system model, actual outputs, noise covariance of process and outputs. Once these are known, Kalman filter recursively average the output corresponding to required unknown state. Step by step development of its algorithm is explained below

1) System Model

The forced system model whose unknown states (x_t) are to be estimated from actual output (z_t) takes the shape

$$x = Fx_t + Gu_t + w_t$$

$$y_t = Hx_t + v_t$$
(1)

where x is the system state matrix of order n x 1, F is the state coefficient matrix of order n x n, x_t is the previous state matrix of order n x 1, G is the input coefficients matrix of order n x m, u_t is the input matrix of order m x 1, w_t is the process noise matrix of order n x 1, z_t is the outputs matrix of order m x 1, H is the output coefficient matrix of order m x 1, H is the output coefficient matrix of order m x 1. Where m denote the number of inputs and n denote the order of the system.

2) Noise Statistics

Kalman filter requires process and output noise covariance to be specified. That is computed from

$$Q = E[w_t w_t^T] \tag{2}$$

where Q is the process noise covariance, and

$$R = E[v_t v_t^T] \tag{3}$$

where R is the output noise covariance.

As Kalman filter implement recursive algorithm, we need to initialize state and error covariance matrix. Afterwards it recursively compute error covariance from

$$P_t = E[e_t e_t^T] \tag{4}$$

where P_t is the error covariance, $e_t = x_t - x_t^{\hat{}}$ is the difference between actual and estimated states, x_t represent actual state matrix and $x_t^{\hat{}}$ represent estimated states matrix.

3) Prior Estimation (Prediction Stage)

After x_{t-1}^{\wedge} and P_{t-1} is initialized, Kalman filter compute $x_t^{\wedge-}$ (i.e. state estimate prior to the occurrence of output) and P_t^- (i.e. error covariance prior to the occurrence of output) using relation

$$x_{t}^{\wedge -} = F x_{t-1}^{\wedge} + G u_{t-1}$$
$$P_{t}^{-} = F P_{t-1} F^{T} + G Q G^{T}$$
(5)

as this computation is performed before actual time has arrived, we call this stage as "Prediction stage".

4) Correction Stage

Now as actual output (z_t) is available, the x_t^{-} and P_t^{-} are validated with available output, necessary scaling of error is done by multiplying K_t ' (Kalman Gain) and is added with reference offset x_t^{-} as below

$$x_t^{^{^{^{^{^{-}}}}}} = x_t^{^{^{^{^{-}}}}} + K_t(y_t - Hx_t^{^{^{^{^{^{^{-}}}}}})$$
(6)

In (ii) K_t must be known to finalize the estimate, which is derived in four steps i.e. Put $x_t^{\hat{}}$ in (i) and then put e_t in $P_t = E[e_t e_t^T]$ and solve for P_t . Now choose K_t so that terms containing K_t are zero and finally solve for K_t to finally arrive at

$$K_t = P_t^{-} H^T (H P_t^{-} H^T + R)^{-1}$$
(7)

Once K_t is computed, putting it in (i) gives the required state estimate of immeasurable state. Finally x_{t-1}° and P_{t-1} matrices are updated to redo the same process.

2.2 Extended Kalman Filter

In case of nonlinear system modeled as

$$x_{t} = f(x_{t-1}, u_{t-1}, w_{t-1})$$

$$y_{t} = h(x_{t}, v_{t})$$
(8)

Estimation becomes a two-step process

- i) Linearization of System
- ii) Implementation of Kalman Filter Algorithm

i) Linearization

In linearization step, Jacobean matrices are computed as

$$A_{[i,j]} = \frac{\partial f_{[i]}(\hat{x_{t-1}}, u_{t-1}, 0)}{\partial x_{[j]}}$$

$$B_{[i,j]} = \frac{\partial f(\hat{x_{t-1}}, u_{t-1}, 0)}{\partial u_{[j]}}$$

$$W_{[i,j]} = \frac{\partial f_{[i]}(\hat{x_{t-1}}, u_{t-1}, 0)}{\partial w_{[j]}}$$

$$H_{[i,j]} = \frac{\partial h_{[i]}(\hat{x_{t}}, 0)}{\partial x_{[j]}}$$

$$V_{[i,j]} = \frac{\partial h_{[i]}(\hat{x_{t}}, 0)}{\partial v_{[j]}}$$
(9)

and then these matrices are evaluated at Linearization point (x_t, u_t) to get the linear form of nonlinear model.

ii). Once linear model is available we apply algorithm mentioned in A'.

3. MIMO TWIN ROTOR SYSTEM

The twin rotor CE-150 is a nonlinear system comprising of seven states. It has two inputs, one each for elevation and azimuth motors. Two outputs of this system are elevation and azimuth angles respectively. The nonlinear model of this system takes the shape

$$\dot{x_1} = \frac{1}{T_1} (-x_1 + u_1)$$

$$x_2 = x_3$$

$$\dot{x_3} = \frac{1}{I_1} ((a_1 x_1)^2) + b_1 x_1 - B_1 x_3 - T_g \sin x_2 - K_{gyro} u_1 x_6 \cos x_2)$$

$$\dot{x_4} = \frac{1}{T_2} (-x_6 + u_2)$$

$$\dot{x_5} = x_6$$

$$\dot{x_6} = \frac{1}{I_2} ((a_2 x_4)^2) + b_2 x_4 - B_2 x_6 - T_{prx_7} - K_r T_{or} u_1)$$

$$\dot{x_7} = -T_{pr} x_7 + K_r T_{or} u_1$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_5 \end{bmatrix}$$
(10)

where $[x_1 x_2 x_3 x_4 x_5 x_6 x_7]^T$ are the main motor speed, elevation angle, angular speed in elevation, side motor speed, azimuth angle, angular speed in azimuth and angular moment caused by

EKF algorithm has been applied to twin rotor system model. Jacobeans for above system before substituting the point of linearization are listed in Eq (11).

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{(b_1 + 2a_1x_1) - T_g - B_1}{I_1 & I_1} & 0 & 0 - K_g u_1 \cos x_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{T_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{(b_2 + 2a_2x_4)}{I_2} 0 & \frac{-B_2}{I_2} & \frac{T_{pr}}{I_2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -T_{pr}. \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{T_1} & 0\\ 0 & 0\\ -K_g x_6 \cos x_2 & 0\\ 0 & \frac{1}{T_2}\\ 0 & 0\\ -K_r T_{or} & 0\\ K_r T_{or} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ D = \begin{bmatrix} 0 & 0\\ 0 & 0\\ 0 & 0 \end{bmatrix}$$

Linearization of this system at the initial point $x_t=0$, $u_t=0$ gives following matrices

Once recursions start, this Jacobean is computed online every time by substituting the estimated values of state matrix x_t^{\uparrow} and u_t .

Note that in (12) the fifth column corresponding to Azimuth angle is zero i.e. this state (and its corresponding output) will not be measurable in open loop. Moreover the states x_1 , x_3 and x_6 are angular/motor speeds that are not practically measurable. These immeasurable states

or any other system state values are closely estimated using EKF here.

4. SIMULATION RESULTS

(Results for self-generated state behaviors)

In this stage states with different behaviors are generated in Matlab (these are taken as actual states) and outputs readings are computed. Outputs are invoked in EKF that yields the estimate of the states. The comparison of actual and estimated state and error between them is presented.

Initialize $x_0 = [0\ 0\ 0\ 0\ 0\ 0]^T$, and $P_0 = I$. Process and measurement noise covariance are updated at every time step.

4.1 Estimation of process states with random behavior

State estimate for x_2 compared with actual state, which is shown in Fig. 1.



Fig. 1: Comparison of Actual and Estimated Elevation Angle when Elevation angle takes random values

The error between actual and estimated state x_2 is shown in Fig. 2.



Fig. 2: Error between actual and estimated Elevation angle when elevation angle takes random values

4.2 Estimation of process states with exponential behavior

Actual states are now taken exponential. Comparison of actual and estimated states is shown in Fig. 3.



Fig. 3: Comparison of Actual and Estimated Azimuth angle when it takes exponential values

4.3 Estimation of process states with sinusoidal behavior

Actual states are now taken sinusoidal. Comparison of actual and estimated states is shown in Fig. 4.



Fig. 4: Comparison of Actual and Estimated Elevation angle when it takes sinusoidal values

4.4 Estimation of process states with abrupt behavior

Actual state of elevation angle is now modeled to take abrupt variations in magnitude. Comparison of actual and estimated states is performed as shown in Fig. 5.



Fig. 5: Comparison of Actual and Estimated Elevation angle when it takes abrupt values

5. Experimental Results

Results for Practical readings of inputs and outputs for Twin rotor System are presented in this section. Following figure shows the experimental setup:



Experimental setup for twin rotor system

Above shown CE-150 twin rotor system is developed by HUMUSOFT is a multidimensional naturally unstable system with two controlled inputs and two measured outputs with significant cross couplings. The Lab CE-150 TRS is designed for the study of dynamical systems, as well as for experiments supporting Control Theory. The experimental results are discussed next.

5.1 Readings in Open loop

The inputs and outputs of the twin rotor system are shown in Fig. 6 and 7 respectively.

Note:For open and closed loop analysis, reading on time axis is taken after every 0.001 seconds of real time (7917 readings were taken in open loop and 5636 readings in closed loop.)



Fig. 6: Two inputs to the Twin rotor system



Fig. 7: Two outputs of the Twin rotor system

The estimate of all the states is obtained. For demonstration estimate of state 'x l and x5' is shown in Fig. 8 and 9 respectively.



Fig. 8: Estimate of state x1



Fig. 9: Estimate of state x5

The comparison of actual and estimated outputs is shown in Fig. 10 and 11.



Fig. 10: Comparison of actual and estimated elevation angle output



Fig. 11: Comparison of actual and estimated Azimuth angle output

5.2 Readings in Close loop

The inputs and outputs of the twin rotor system are shown in Fig. 12 and 13. respectively.



Fig. 12: Two Controlled inputs of the Twin rotor system



Fig. 13: Two outputs of the Twin rotor system

The estimate of state 'x1and x6' computed using EKF is shown in Fig. 14 and 15 respectively.



Fig. 14: Estimate of state x1



Fig. 15: Estimate of state x6

The comparison of actual and estimated outputs is shown in Fig. 16 and 17.



Fig. 16: Comparison of actual and estimated elevation angle output



Fig. 17: Comparison of actual and estimated elevation angle output

6. Conclusion

The simulation tests for EKF were run by assigning different behaviors to the twin rotor states. Successful state estimation was observed for random, exponential, sinusoidal and abrupt state behaviors. EKF state estimates quickly converged to the actual values of the state. After convergence, error between actual and estimated states was minute that exhibit the high accuracy of EKF algorithm. Afterwards, states estimate (including immeasurable states) were extracted from the experimental data for open and closed loop twin rotor using the EKF. These estimates serve as an alternate to a highly precise and costly instrument that was not available to measure the states.

7. References

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