A Novel Distributed Load Scheduling Algorithm for Electrical Vehicles in Smart Grid

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Abstract

This paper explains the implementation of a load scheduling technique developed to schedule the electric vehicles (EVs) charging and discharging using optimal control in the distributed power network. We have assumed that the power flow between EVs and the utility grid is multi-directional, i.e., EV can supply power to grid in reverse as the distributed energy source as well as it could be charged from the grid, this phenomena is known as vehicle to grid (V2G) system. The optimal load scheduling algorithm has been developed as a varied discrete program (VDP) constraint, which is NP-hard and extremely complex to compute. Therefore, to overcome this complexity we have proposed a technique to compute the VDP constraint by altering the basic load profile curve of the night hours, and designed a distributed algorithm to reiterate water-filling. This algorithm executes in the distributed manners and effectively schedules EVs charging/discharging in local areas where the aggregation structure is already established. Owner’s security enhancement and reduced computational load are the major benefits of our proposed algorithm; the performance of this algorithm is verified by simulations.

Key Words: Smart grid, Distributed Control, Load Diversion, Optimum Load scheduling, Electric vehicles

1. Introduction

Due to massive release of greenhouse gasses (GsG) and consistently depleting fossil fuel reserves, it is anticipated that the survival of future generations will be very difficult due to growing ecological pollution and without conventional energy resources. Therefore, it is considered that renewable resources and electric vehicles (EVs) are the best alternative of conventional power generation means and internal combustion engine vehicles (ICEVs) [1]. Despite of severe atmospheric disadvantages, the fossil fuels are more expensive than electricity, which immensely affects the economy of developing countries. It is projected that EV numbers will increase rapidly in the near future, because it exhibits better efficiency than a ICEV car [2].

Although, the growing use of EV will severely affect the reliability of the power grid. The random EVs charging could increase the load in peak hours, or it can disturb the prevailing power demand. Consequently, it can cause severe destruction within the power grid, as well as damaging the household appliances [3]. Conversely, by using the demand side management (DSM) technique we can intelligently schedule EVs charging by considering the needs of each stakeholder connected to the power grid. This can be achieved by the installation of autonomous DSM based charging stations, advance metering setups, and by deploying efficient communication means [4].

Massive research has been carried out to upgrade EVs interior for DSM collaboration. It depends on which direction the power is flowing between EVs and grid. The contemporary carried out research might be divided into two classifications, 1) accommodation of those EVs who are just charging and only availing grid to vehicle (G2V) facility, 2) research about facilitating those EVs who are using both G2V and V2G facility. In [5]-[10], multiple algorithms have been formulated to schedule EVs charging to implement G2V. In [5], researchers created a charging constraint as limited time horizon vibrant game, and proposed a scattered charging method.

In [6] an inclusive investigation for load shifting constraints has been carried out using EVs, in this research, an optimal distributed algorithm has been proposed. In [7]-[9], multiple concept of distributed algorithm are illustrated, mainly use non-centralized water filling technique, to stabilize the demand profile for low power transformers to attain the best scheduling for EVs charging. In these approaches, network controlled EVs have been penetrated to regulate the grid supply in an American state with the mixture of renewable resources using the phenomena of consistently
varying electricity prices [10]. In [11], autonomous power inverters for V2G enabling EVs are used to supply energy in return to the grid, this study allowed researchers to deploy both V2G and G2V systems. In [12], authors are intended to reduce the EVs aggregation cost. Therefore, they have proposed an optimal load scheduling algorithm for both global and local load management, which can efficiently manage the random charging of multiple EVs beside it can level the load curve for conventional households. In [13], an optimal programming based charging/discharging scheduling algorithm is proposed, which controls uncertainties of renewable power production in smart grid. In [14], a stochastic process is deployed to reduce the known management cost of the grid considering the arbitrary EVs actions. In [15], researches regulated the overall load demand of multiple households along with managing the entire grid using EVs by deploying efficient load aggregation models. In [16], a dynamic central load scheduling technique is proposed to simultaneously manage the EVs and household power consumption, which enhances the sustainability and reliability of the smart grid.

A smart grid is a vibrant and consistently expanding network, which could not be controlled by penetrating a centralized algorithm, because it is impossible to attain exact information about each and every part of the power grid in real time, which usually occurs due to massively growing renewable penetration, therefore the scalability and information analysis of the global power networks is becoming difficult. Thus, the distributed management is the alternate solution to overcome these constraints [17]. By using modern technologies it is leniently possible to deploy distributed algorithm to manage the grid, this grid managing approach is also enforced by various researches. Normally, besides the viability and advantages of broader implementation of optimization techniques, the incentives of distributed algorithm have been analyzed by comparing the results with centrally optimized networks to check the robustness, reduction in communication burden, optimization of the load constraints, as well as enabling the stable flow of energy between stakeholders [18].

In articles [19] and [20], authors have proposed different models for controlling grid operations and introduced charging scheduling schemes for battery swapping stations. As analyzed, although in these studies the optimization constraints related to real time and day-ahead scheduling have been effectively analyzed and these research studies have successfully developed the charging optimization techniques suitable for single charging stations or for a solo aggregator, but the optimization scheme for interactively connected numerous charging stations is missing. In ref [21] authors have developed an optimal scheduling algorithm for distributed EVs charging by deploying valley filling technique, and in article [22] author further expended the deployment of electric vehicles as a demand side management tool, which is generally considered as future of the power grids optimization where higher penetration of renewable power generation is anticipated. In ref [23], researcher proposed a local power managing scheme and defined an optimization strategy which efficiently coordinates with scattered EVs and manage their charging and it also controls the operation of battery switching stations. In ref [24], authors identified a robust charging constraint for plugged-in E-taxis which are anticipated as future of public transportation this constraint contains time anecdotal profits. In ref [25] a real time charging scheduling scheme for managing EVs aggregation process has been proposed to make EVs a permanent part of the energy markets.

In ref [26], [27], authors mainly focused to model schemes used to control uncertainties of the day-ahead markets while assuming that the short-term price forecasts in real time markets are precise. Although, these EVs charging scheduling schemes and optimization problems ignore the existing grid models and neglect their operational constraints, therefore, achieved results are potentially impractical. Practically the optimal EVs charging scheduling problems for mega charging stations could be formulated as an optimum-power-flow (OPF) problem having diverse objectified functions in a power distribution network to precisely counter the operational constraints. In [28] a study about the impact of vehicle to grid integration on voltage management in the existing distribution grids has been carried out. In [29], author propose an EV charging scheduling problem as a retreating horizon optimization model, the main objective of this model is to divert the EV charging from peak to off-peak hours. Several optimization model for instance linear optimization model which uses linear programming has been proposed in ref [30], in addition the particle swarm optimization has been proposed in [31] which work efficiently while coping network optimization constraints. Ref [32], [33], presents the most recent studies about convex relaxation models used to formulate the OPF optimization problem under precise relaxing scenarios. The following relaxation models are able to generate concise solutions for the actual formulations which are
globally optimal and have the frequent computational capabilities, especially for those distribution networks where relaxation accuracy could be achieved leniently.

The highly esteemed success and fame of EVs has been anticipated in the upcoming decade, in this research, we have developed an autonomous load diverting method for night hours by employing both V2G and G2V enabling EVs. However, previous work, i.e. [12], also illustrates a distributed load scheduling technique by penetrating both V2G and G2V concepts, we appreciate these researchers approach, since this research article is the first one introduces the concept of stabilizing the load demand curve by utilizing optimum solutions, it configures the multi-directional power dispatch through decentralized algorithm. The main contributions of our research paper are as follows;

- We have developed an optimal load shifting constraint as VDP constraint; the basic objective of this constraint is to stabilize the load curve accordingly satisfying the EV charging and discharging requirements. By considering the NP-hardness of the optimal VDP constraint, we have proposed a robust and leniently computable constraint estimation technique which efficiently forecasts the nominal load demand profile curve for the night hours. The actual objective of this load profile estimation constraint is to divide the total time of a day into two distinct time slots through optimum time brinks, after implementing this method an EV will only charge or discharge following the defined schedule.

- In order to efficiently execute the load profile estimation constraint, we have proposed an optimal distributed load scheduling algorithm, which uses the bilateral water filling technique for load leveling. It is distributed in such manners that an EV performs local load calculation itself and individually communicates this information to the central control station, in order to accommodate the aggregation process which also reduces the computational burden. We have illustrated the brief convergence analysis of the proposed model in upcoming sections.

- The simulation results show that our proposed algorithm can efficiently anticipate the nominal load demand profile for night hours, even when it is projected that multiple EVs would be simultaneously charging, moreover it can also define charging and discharging schedules for EVs and it also ensures the information privacy for each EV.

To prove relevancy and effectiveness of the proposed algorithm, the simulation results have been compared with the existing BONMIN method, the intensive performance comparison shows that our proposed algorithm exhibits extremely fast computation speed. In addition extremely complex research comparisons proves that, as compared to the BONMIN load scheduling method our algorithm can anticipate the load demand curve within seconds while BONMIN requires hours to perfume similar task.

This paper is arranged in the following manners; Section II, covers the single EV optimization model, network formulation, and defines the distributed water filling algorithm. Section III, explains the estimation of the primordial optimization, constraint, and illustrates a distributed optimal load scheduling algorithm. Section IV presents the simulation results, and discusses the performance analysis of the proposed algorithm, whereas section V concludes this paper.

2. Mathematical Model Formulation

This section covers the initial formulation of the dynamic model framework for an individual EV and explains the basic structure of the proposed constraint besides, the detailed description of the distributed water filling based load scheduling technique is also given in this section. Note that, actually this model is the extension of our previously proposed research work [4], [5].

2.1. Dynamic Model Framework for an Individual EV

We presume that each EV contains Lithium-ion battery modules which are capable enough to perform V2G and G2V operations. Assume \( t = 0, 1, \ldots \) represents the time period containing multiple sampling intervals \( \Delta T \). The battery state of charge (SOC) at time interval \( t \), given by \( e(t) \) (%), it also denotes the current charge state of a battery and defined as;

\[
e(t) = \frac{G(t)}{G_{\text{max}}} \tag{1}
\]

where \( G_{\text{max}} \) is the overall battery capacity in (kWh), and \( G(t) \) is the current charging state of a battery at time \( t \).

Assume \( a(t) \) represents the energy flow from EV to grid in kW. Let’s presume that the...
charging/discharging ratio of an EV is stable as  
\( a(t) \) is in the hiatus form, from 0 to 0.  
\( a(t) \geq 0 \) it shows that an EV is being charged at time  
\( a(t) < 0 \) denotes the EV discharging. Moreover \( a(t) = 0 \) denotes the absence of energy  
flow from EV to the grid. Thus, we can get the following unequal problem;

\[-a^{-\text{out}} \leq a(t) \leq a^{-\text{in}} \quad (2)\]

where \( a^{-\text{in}} \) is the highest power supplied for  
charging an EV, while \( a^{-\text{out}} \) is the highest  
discharging ratio of an EV battery. For G2V  
operation let, multiple EVs are connected to  
intelligent chargers, which are capable enough to  
alter EV numbers from 0 to \( a^{-\text{in}} \), conversely, for  
V2G operation; various EVs are connected to  
intelligent inverters, which exhibits the capability  
of supplying energy to the grid from EVs with the  
ratio of 0 to \( a^{-\text{out}} \).

We characterize 0 \(< \eta_{\text{in}}, \eta_{\text{out}} < 1 \), in order to  
represent the power supplying efficiency of an  
EV during charging/discharging, so that we get the  
updated SOC as;

\[\eta(t) = \begin{cases} 
\eta_{\text{in}} < 1, & \text{for } a(t) \geq 0. \\
\eta_{\text{out}} > 1 & \text{for } a(t) \leq 0. 
\end{cases} \quad (3)\]

Moreover, to extend the battery life, it is  
proposed that the SOC should remain within 25%  
out of 90% range [33]. Assume that an EV shall be  
disconnected at time interval  \( T \), then we get  
following problem.

\[e \leq e(0) + DT \sum_{t=0}^{T-1} \eta(t) a(t) \leq \bar{e}, \quad \tau = 1, \cdots, T \quad (4)\]

where  \( a(0) \) is the initialization of the EV charging  
outlets at time 0. We have restricted the EV battery  
discharging and charging at  
\( e = 25\% \) to  \( e = 90\% \).

An EV owner is capable to select its  
preferred SOC represented by  \( e^* \) this is the desired  
SOC at time slot  \( T \). It should be noted that  
\( e^* \in [e, \bar{e}] \). In this case we can get the following  
homologous problem.

\[e(0) + DT \sum_{t=0}^{T-1} \eta(t) a(t) \frac{G_{\text{max}}}{G_{\text{max}}} = e^* \quad (5)\]

2.2. Constraint Initialization

In this research, we have only considered  
EVs having the capability of both V2G and G2V  
operations to stabilize the nominal load profile  
curve during the night hours. For simplification,  
we presume that all EVs connect to the grid at 8  
pm (20:00) and disconnects at 8 am in the next  
morning. The nominal demand curve of common  
households (excluding EVs) from 8 pm to 8 am is  
given in Fig. 1.

The basic power demand is represented by  
\( P(t), \forall t = 0 \cdots T − 1 \). Assume that there  
are  \( c \) number of EVs, characterized from 1 to  \( c \). The general specifications of an EV as well as the  
charging/discharging amount variables have been  
characterized by  \( d = 1 \cdots c \), where  \( d \) illustrates the  
\( d^{\text{th}} \) EV. The variable  \( X \) represents the common  
structural specifications of an EV fleet. The  
op tum load diverting constraint has been  
initialized as presented.

\[\min \sum_{t=0}^{T-1} \left(P(t) + \sum_{d=1}^{c} a_d(t)\right)^2 \quad (6)\]

- Repression
  - Impartiality repression on  \( a_d(t) \)
    \[e_d = e_d(0) + DT \sum_{t=0}^{T-1} \eta_d(t) a_d(t) \frac{G_{\text{max}}}{G_{\text{max}}} \]
    \[= e_d^*, \quad \forall d. \quad (7)\]
  - disparity repression on  \( a_d(t)\)
    \[-a_d^{-\text{out}} \leq a_d(t) \leq a_d^{-\text{in}}, \quad \forall d, t \quad (8)\]
  - disparity repression on  \( e_d(t)\)
    \[e \leq e_d(t) \leq \bar{e}, \quad \forall d, t. \quad (9)\]
  - discrete repression on  \( \eta_d(t) \)
    \[\eta_d(t) = \begin{cases} 
\eta_{d,\text{in}} < 1, & \text{for } a_d(t) \geq 0, \\
\frac{1}{\eta_{d,\text{out}}} > 1 & \text{for } a_d(t) < 0, 
\end{cases} \quad (10)\]

Assertion 1: We have analyzed that the particular  
load scheduling constraint which has been executed  
using formulations (6)-(10) is the optimization  
integer which does not only consider  \( a_d(t) \) besides  
it also inherits  \( \eta_d(t) \). Since,  \( \eta_d(t) \) can only  
be inherent discrete values, whereas  \( a_d(t) \) could work on  
continuous values, note that the formulations (6)-(10)  
are the VDP constraints. In total there are just 3\( cT \) optimization integers.  
Although, if we only consider charging constraint,  
i.e. the task completion in [5], then  \( \eta_d(t) \) is a  
constant integer, parallel to  \( \eta_d^{\text{in}} \), which is not  
permitted to change.
Our objective is to manage the optimum load shifting in a distributed network with the help of aggregator. It is assumed that the aggregator is forecasting the basic load demand before 24 hours. The communication medium between aggregator and EVs is the star network, in which the aggregator has the central node, while EVs are connected through tree leaf nodes. All EVs perform two way communications with the aggregation center. By implementing our proposed algorithm an aggregator will have the leverage that it is not compulsory to be computationally strong, since the computational burden will be consistently shared with the EVs which are scattered. Moreover, as aforementioned it is not essential for an aggregator to have all the specifications information about each EV, so that the owners’ privacy is ensured.

### 2.3. Water Filling based Decentralized Algorithm

Before illustrating the proposed algorithm for scattered EVs scheduling, initially we will enlighten our previously proposed algorithm which works on water filling technique [5].

Diverse from (2), the disparity repression used for EVs in [5] is as follows;

\[ 0 \leq a_d \leq a_d^{in} \quad \forall d, t. \]

**Fig 1:** Basic load profile of 100 house-holds served in California Edison region from 20:00 on September 10, 2018 to 8 am September 11, 2018 [24]

In case, if owners have leverage to select SOCs according to their own requirements, then the Impartiality repression is equal to (5).

\[ e_d(0) + \eta_d^{in} \Delta T \sum_{t=0}^{T-1} a_d(t) \leq e_d^*, \quad \forall d. \]

The tariff function is similar to (6), and the integral constraint initialization is represented as:

\[ \min \sum_{t=0}^{T-1} (P(t) + \sum_{d=1}^{c} a_d(t))^2, \text{ e.t. } 0 \leq a_d \]

\[ e_d(0) + \eta_d^{in} \Delta T \sum_{t=0}^{T-1} a_d(t) \leq e_d^*, \quad \forall d. \quad (11) \]

Defining the projected map

\[ \Gamma_d[r] = \begin{cases} a_d^{in} & r > a_d^{in} \\ 0 & 0 \leq r \leq a_d^{in} \\ 0 & r < 0 \end{cases} \quad (12) \]

**Algorithm 1: Distributed Water Filling**

**Requisites:** \( a_d^{in}, e_d(0), e_d^*, \in \) and \( P(t), \forall d, t; \)

**Assure:** \( a_d(t), \forall d, t; \)

1. The expected total load forecast shared with aggregator \( P(t); \)
2. for \( d = 0, 1, 2, \ldots, c \) do
3. Each aggregator calculates \( P_d(t) = \sum_{s=1}^{d} a_s + P(t), \forall t, d \geq 2; \quad P_0 = P(t), \forall t, d = 1; \)
4. Aggregator transmits \( P_d(t) \) to the \( d \)’th EV;
5. For \( d \)’th EV;
6. Boot \( \tilde{\gamma}_d = a_d^{in} + \max_d P_d(t) \) and \( \gamma_d = \min_d P_d; \)
7. while \( \tilde{\gamma}_d - \gamma_d > \epsilon \) do
8. execute \( \gamma_d = (\tilde{\gamma}_d + \gamma_d)/2; \)
9. run \( a_d = \Gamma_d[\gamma_d - P_d(t)] \) and \( \sum_{t=0}^{T-1} a_d(t); \)
10. upgrade \( \tilde{\gamma}_d \) and \( \gamma_d \) accordingly
\[ \tilde{\gamma}_d = \gamma_d \quad \gamma_d = \gamma_d \\
\]
11. end while
12. the \( d \)’th EV forwards \( a_d(t) \) to aggregator;
13. end for

Since, in the case of smart grid network multiple algorithms are being used to reduce the chance of complete system failure. We will redefine the algorithm proposed in our prior research work [5] (Algorithm 1).

The eventual water level is represented by \( \gamma^* \), in fact it has been used to stabilize the \( \gamma^* = -\lambda^* \), here \( \lambda^* \) is the optimum Lagrange multiplier. The altering \( \epsilon \) has been represented as the tolerating difference between higher and lower water level that is the tiny positive number. Initializing \( H \) as the figure of bisectonal requirement, we get:

\[ \epsilon \geq 2^H \left( \tilde{\gamma} - \gamma \right), \quad (13) \]

or

\[ H \geq \log_2 \left( \frac{\tilde{\gamma} - \gamma}{\epsilon} \right) \quad (14) \]
From (14) it could be evaluated that a higher \( \varepsilon \) intended to produce a severe deviation to the eventual result, conversely it minimizes the computing period. Regardless of this, a tiny \( \varepsilon \) intends to rise the computing period, and it increases the preciseness of the proposed algorithm. Hence, the strength of \( \varepsilon \) depends on the accuracy of different scenarios. In a particular case where accuracy is the main priority of an algorithm as compared to its efficiency, a higher \( \varepsilon \) is selected respectively for more details visit [5].

**Assertion 2:** In [4] we execute the load managing constraint by using the charging mechanism of EVs. The EVs are connected to smart charging points which are only able to accomplish G2V task. In such scenario, in [5] just valley filling (instead of peak shaving) has been accomplished. The algorithm 1 is denoting the water filling model due to the eventual result which seems much similar to the natural concept of water filling, here \( P(t) \) is genuinely water surface, whereas \( \gamma_c \) is the resultant water level after water filling.

### 3. Results

The following section explains the estimation of the primal optimization, constraint (6)-(10), which is relative to the proposed distributed scheduling algorithm; in addition, we have also defined the overview of the convergence capability of our proposed algorithm.

#### 3.1. Constraint Estimation

This subsection illustrates the estimation of the primal VDP constraints (6)-(10). Before going into the detailed analysis, initially we examine the problems faced by the primal optimization constraints (6)-(10) by directly computing the projected load profiles.

- The optimization integer \( a_d(t)' e \) within the primal optimization constraints (6)-(10) is firmly attached, the main reason of the following firm attachment is the disparity constraint (9). The simultaneous charging/discharging of multiple EVs forced us to overcome the problems which are related to SOCs. Although whenever the alterations are the main focus, then the constraint (9) turns into redundant, which enable us to solve equation (11) leniently.
- The optimization integers \( \eta_d(t)' e \) in the primal optimization constraint (6)-(10) executes during receiving discrete values. To resolve the complication caused by the discrete integers \( \eta_d(t)' e \); we define the constraints (6)-(10) in such manners, that these formulations could also be analyzed consistently as confined integrating nonlinear programming (CINLP) constraint, which is NP hard [36]. Further, \( \eta_d(t)' e \) depends on \( a_d(t)' e \), hence these are also firmly attached.

From the aforementioned description, it has been proved that it is enormously complex to directly compute constraints (6)-(10), nor these constraints could be solved using decentralized approach, thus these analytics compel us to search for a confined and computable estimation for the constraints (6)-(10). Since, we are mainly focusing on flattering the load profile estimation during the night hours. Therefore, we recall the statistics of Fig. 1, which helps us to evaluate the following core features of the basic load curve.

- 20:00 is the peak demand time.
- The basic load demand reduces monotonously till 4:00 am.
- Whilst the basic demand is minutely raised from 4 to 8 am.

Several types of load demand data have been collected from different authorities and institutions, i.e., the real time data in hourly slots have been collected from British ISO [34], [35]. In addition, the data collected from Midwest ISO [36] has shown the same results as shown in Fig. 1. Hence, it is imperative to explain that the basic load demand curve, without EVs in night hours flattens the given assumptions as follows;

**Assumption 1:** \( P(t) \) is monotonously enhancing \([0, t_y] \) and reaches at its peak value at time instance \( t_y \). Moreover, \([t_y, t_z] \), \( P(t) \) is monotonously declining, and gains its highest value at time instance \( t_z \). In \([t_z, T] \), \( P(t) \) which is monotonously rising. It should be clear that, in Fig.1. the \( t_y = 0 \) and \( t_z \) denotes the lowest load demand.

The G2V ability of an EV makes valley filling possible, whereas V2G make peak shaving possible. It could be interpreted as, we can synchronize EVs in such manner that EVs supply power to the grid during those periods when the basic load demand is at its peak, and it charges in those hours when the basic load demand is in valley. By such coordination, the load profile curve has been stabilized. Since, it is usual that the maximum load demand rises before the valley demand, therefore we are intended to compute the threshold \( t^* \), so that EVs discharges at \([0, t^*] \) and charge at \([t^*, T] \) after that it computes the optimum load scheduling for EVs charging/discharging.
We divide $X$ into two subsets $X_1$ and $X_2$, where $X_1 = \{d : e_d(0) > e_m\}$, and $X_2 = \{d : e_d(0) \leq e_m\}$. After recalling our previous evaluation given in [5], we suggest following estimations for the primal constraints (6)-(10).

$$\min \sum_{t=0}^{T-1} \left( P(t) \sum_{d=1}^{c} a_d(t) \right)^2$$

\text{e.t.} - a_d^{\text{out}} \leq a_d(t) \leq 0, \forall t = 0, \ldots, t^1 - 1, d \in X_1,

$$a_d = 0, \forall t = 0, \ldots, t^1 - 1, d \in X_2,$$

$$0 \leq a_d(t) \leq a_d^{\text{in}}, \forall t = t^1, \ldots, T - 1, d,$$

$$e_d(0) + \Delta t \sum_{t=0}^{T-1} \eta_d(t) a_d(t) \frac{G_d^{\text{max}}}{1} = e_m, \forall d \in X_1,$$

$$e_d(0) + \Delta t \sum_{t=0}^{T-1} \eta_d(t) a_d(t) \frac{G_d^{\text{max}}}{1} = e^*_d, \forall d,$$  \hspace{1cm} (15)

where $e_m$, $t^1$, $\eta_d(t)'e$, and $a_d(t)'e$ are the optimization integers, and $e_m$ is interpreted as the required SOC after full discharge, which is often shared by each EV integrated within a EV fleet. Alongwith for EVs, the available subset $X_2$ never discharges at $[0, t^1]$, since this subset is only concerned about the current state of charging prior to $t^1$.

It should be noted that, in e.q (15), $\eta_d(t)'e$ and $a_d(t)'e$ are different from the primal optimization constraints (6)-(10) in the following problems only. The disparity problem (9) are not present in (15), making (15) lenient to compute as compared to the equations (6)-(10).

### 3.2. Distributed Load Scheduling Algorithm

In this subsection we have proposed a distributed load scheduling algorithm for optimum load diversion constraint by employing EVs charging/discharging, this algorithm works on the distributed water filling technique. The basic approach is to compute $t^1$ and $e_m$ in the distributed prospective, after that $a_d(t)'e$ is readily achieved. The computing procedure of our proposed algorithm is explained next, in which for initialization we have assumed $e_m = \varepsilon$.

**Step 1:** All EVs employ the distributed water filling technique to compute the following constraint.

$$\min \sum_{t=t^1}^{T-1} \left( P(t) + \sum_{d=1}^{c} a_d(t) \right)^2$$

\text{e.t.} 0 \leq a_d(t) \leq a_d^{\text{in}}, \forall t = 0, \ldots, T - 1,

$$e_m + \eta_d^{\text{in}} \Delta T \frac{\sum_{t=t^1}^{T-1} a_d(t)}{G_d^{\text{max}}} = e_d^*, \forall d \in X_1,$$

$$e_d + \eta_d^{\text{in}} \Delta T \frac{\sum_{t=t^1}^{T-1} a_d(t)}{G_d^{\text{max}}} = e_d^*, \forall d \in X_2,$$  \hspace{1cm} (18)

That represents the optimal solution for (18) through $a_d^{\text{in}}'e$. After executing these constraints, the aggregator will receive a time slot $t^{in}$ so that,

$$a_d^{\text{in}}(t) = 0, \forall t < t^{in}, \text{and} a_d^{\text{in}}(t^{in}) > 0, \forall d,$$  \hspace{1cm} (19)

where we have an executing water level which is represented by $\gamma^{\text{in}}$, and defined as;

$$\gamma^{\text{in}} = P(t^{\text{in}}) \sum_{d=1}^{c} a_d(t^{\text{in}})$$
Step 2: Since, only EVs are available in the subset $X_1$. Therefore, we are activating the discharging state. Assume those EVs which are included in the subset $X_1$ use the distributed water filling technique to compute the following constraint:

$$
\min \sum_{t=0}^{T-1} \left( P(t) + \sum_{d \in X_1} a_d(t) \right)^2
$$

e. t. \quad a_d^{\text{out}} \leq a_d(t) \leq 0, \quad \forall t = 0, \ldots, T - 1, d \in X_1
$$

where we have fixed $\epsilon_{\text{min}} = \epsilon$. It represents the optimum solution of the constraint (20) through $a_d^{\text{out}} = \epsilon$. For EVs in $X_2$, fix $a_d^{\text{out}} = 0, \forall t$. After converging the aggregator we achieve a time instance $t^{\text{out}}$ so that:

$$
a_d^{\text{out}}(t) = 0, \forall d > t^{\text{out}}, \text{and } a_d^{\text{out}}(t^{\text{out}}) < 0, \quad \forall d \in X_1
$$

Step 3: In this stage, a local EV aggregator compares two different water levels. By assumption 1, we get satisfactory $t^{\text{in}}$ and $t^{\text{out}}$, which will descend during the intervals $[t_y, t_z]$. Due to declining monotonous, either evaluation $y^{\text{out}} - y^{\text{in}}$ is similar to this evaluation if $t^{\text{out}} \leq t^{\text{in}}$.

$$
t^+ \in [t^{\text{out}}, t^{\text{in}}], \quad \epsilon_{\text{min}} = \epsilon
$$

and

$$
a_d(t) = a_d^{\text{out}}(t) + a_d^{\text{in}}(t), \quad \forall t, d.
$$

Some of the details of the proposed algorithm are explained here.

- The optimally regulated load demand profile will be in a straight line which is not possible in practical case, since, the rate of discharging and charging of an EV is limited, as well as only few number of EVs usually penetrates with the grid for peak shaving. However, by implementing optimum techniques for scheduling EVs charging/discharging, it is possible to regulate water levels, when if EVs are in discharging state which usually occurs in minimum rate as compared to the charging state, then $y^{\text{out}} \geq y^{\text{in}}$ should be maintained.

- In Stage 3; if $t^{\text{out}} > t^{\text{in}}$, then we can assume that EVs are able to supply enough power back to the grid which can be used for peak shaving. Therefore, in this scenario we should enhance $\epsilon_{\text{min}}$ so that $y^{\text{out}} = y^{\text{in}}$, thus, we get $\epsilon_{\text{min}}$ by the adaptation of vibrant bisection in the distributed network prospective.

Assertion 4: the proposed algorithm in this research work is actually depending on the water filling technique implemented in our previous paper [5], it is leniently established that this algorithm works in a distributed manner. Moreover, on one side our distributed algorithm minimizes the computational load of an aggregator, while on the other side it maintains the owner’s privacy. Therefore, it is not essential for an aggregator to know about each EV specifications.

Assertion 5: To persuade assumption 1, there is a need of extremely high amount of load, so that the proposed algorithm can manage aggregator performance effectively, hence the load of each household can be managed properly by strictly following the specified load scheduling outline, it is also analyzed that if an individual household performs unsatisfactory or it suddenly get out of business, even then it cannot influence the aggregator performance. In such scenario we assume that the assumption 1, could not be contented for fewer load amounts.
For instance, it is anticipated, that the load demand on a micro grid during night hours could have various peak spikes and valleys. Therefore, to tackle these situations, the load demand curve could be divided into various sub demand curves, and the phase targeted SOCs are assigned in such manners that each assignment is optimum and specifically targeting a phase for SOC accomplishment, therefore, this sub demand curve should be equal to the required SOC within a complete curve. Hence, the proposed algorithm could have been implemented for each individual sub curve to achieve the optimal solution for the whole curve.

3.3. The Convergence Evaluation

In this subsection we will have explained the analysis of the convergence results of our proposed scheduling algorithm.

Lemma 1: Let $t^{in}$ and $t^{out}$ exists in such scenario that the conditions (19) and (21) holds when $t^{out} \leq t^{in}$, after that the proposed algorithm converges to the optimum solution for the constraint (15).

To prove theorem 1; the following lemmas are required. Lemma 1 is related to the optimal water filling by penetrating a single EV.

$$\min \sum_{t=0}^{T-1} (P(t) + a(t) - \omega)^2, \quad e.t \quad 0 \leq a(t) \leq a^{-in}, \; \forall t$$

$$e(0) + \eta^{in} \Delta T \sum_{t=0}^{T-1} a(t) / G^{max} = e^{*}$$

(22)

It is interpreted as;

$$a^{*}(t) = \Gamma[-\lambda^{*} - P(t)], \; \forall t,$$

where $\omega$ denotes the optimally aggregated load demand, which remains stable for all $t$, and $a^{*}(t)$ is the optimum solution for the constraint (22), where $\lambda^{*}$ represents the optimum Lagrange multiplier, note variable $\Gamma[\cdot]$ is presented in e.q., (12).

Lemma 2: The optimum solution for the constraint (22) is not considered as the optimal load demand $\omega$.

Proof: we comprise

$$\sum_{t=0}^{T-1} (P(t) + a(t) - \omega)^2 = \sum_{t=0}^{T-1} (P(t) + a(t))^2 + 2\omega \sum_{t=0}^{T-1} a(t) + T\omega^2$$

(23)

As narrated impartiality constraint at $a(t)'e$ exhibits the following condition as follows;

$$\sum_{t=0}^{T-1} a(t) = (e^{*} - e(0))G^{max} / (\eta^{in} \Delta T)$$

(23)

it is a stable constraint. Thus, by merging (23) and (24), we get

$$\sum_{t=0}^{T-1} (P(t) + a(t) - \omega)^2 = \sum_{t=0}^{T-1} (P(t) + a(t))^2 + \rho(\omega),$$

while

$$\rho(\omega) = 2\omega (e^{*} - e(0))G^{max} / (\eta^{in} \Delta T) + T\omega^2,$$

Consequently $\omega$ only regulates the stable terms of the tariff function, thus the Lemma 2 has been proved.

- Now we will prove Lemma 1.

Validation of the Lemma 1; initially we delineate

$$g_{t}(a(t), \omega) = \left( P(t) + \sum_{d=1}^{c} a_{d}(t) - \omega \right)^2$$

$$G(a, \omega) = \sum_{t=0}^{T-1} g_{t}(\omega).$$

It is followed by the tariff equation (6) which is similar to $C(a^{0}, 0)$. Subsequent to the convergence of the proposed algorithm; there would be 2 types of results;

A: Initially we prove that $t^{\dagger}$ is the optimum solution despite having some contradictions. We assume that, there is a possibility of $t^{\dagger} \lt t^{out}$ and $t^{\dagger}$ which establishes that the overall tariff is fewer than $t^{\dagger}$ has, it is represented by $a_{d}^{\dagger}(t)$ which has the optimum solution for $t^{\dagger}$. As given in Lemma 2, we change the preliminary cost function by $G(a, \gamma^{out})$ despite imposing any replacement to the optimum solution. The charging and discharging procedures could be divided into three sections respectively.

- In case of $t \in \{0, t^{\dagger}\}$, we compute constraint (16) by employing our algorithm 1, in which we have replaced $t^{\dagger}$ by $t^{\dagger}$. It denotes the resultant subsequent water level by $\gamma^{out}$. Since it is relative to the natural contrary water filling function, $t^{\dagger} \lt t^{\dagger}$ which leads to $\gamma^{out} \lt \gamma^{out}$, because we have shrieked the EVs discharging duration, when it is still possible to implement load leveling constraint. from Lemma 1; there should be some $t$ left for $a_{d}^{\dagger} \gt -a_{d}^{-out}$ due to this shrieking, in such scenario we get;
\begin{align*}
&\left\{ \begin{array}{ll}
a^+_d(t) = -a^-_{d,\mathrm{out}}, & \text{if } a^*_d(t) = -a^-_{d,\mathrm{out}} \\
a^+_d(t) < a^*_d(t), & \text{if } a^*_d > -a^-_{d,\mathrm{out}}
\end{array} \right. \\
&\text{which leads to } t \in \{0, t^+\},
\end{align*}

\[ P(t) + \sum_{d=1}^{c} a^+_d(t) \leq P(t) + \sum_{d=1}^{c} a^*_d(t), \]

here the impartiality just stands at \( t \) when \( a^*_d(t) = -a^-_{d,\mathrm{out}}(t) \). Further, we also have the information that some \( t \) exists in this case:

\[ P(t) + \sum_{d=1}^{c} a^+_d(t) - \gamma^{\text{out}} = 0 \]

\[ P(t) + \sum_{d=1}^{c} a^*_d(t) - \gamma^{\text{out}} < 0, \]

So that we get

\[ \sum_{t=0}^{t^+} g_t(a^+(t), \gamma^{\text{out}}) > \sum_{t=0}^{t^+} g_t(a^*, \gamma^{\text{out}}), \tag{24} \]

- In case of \( t \in \{t^+, t^\in\} \), we gain \( -a^*_d(t) \leq a^+_d(t) < 0 \) moreover \( a^+_d(t) = 0 \). since \( P(t) + \sum_{d=1}^{c} a^+_d(t) = \gamma^{\text{out}} \), thus it is followed by:

\[ \sum_{t=t^+}^{t^\in} g_t(a^+(t), \gamma^{\text{out}}) \]

\[ > \sum_{t=0}^{t^+} g_t(a^*, \gamma^{\text{out}}), \tag{25} \]

- In case of \( t > t^\in \), due to \( t^+ < t^\in \), as mentioned in Lemma 1, we get \( a^+_d(t) = a^*_d(t) \), which is followed by:

\[ \sum_{t=t^\in}^{t-1} g_t(a^+(t), \gamma^{\text{out}}) \]

\[ > \sum_{t=t^\in}^{t-1} g_t(a^*, \gamma^{\text{out}}), \tag{26} \]

By the equations (25)-(27) we get:

\[ G(a^+, \gamma^{\text{out}}) > G(a^*, \gamma^{\text{out}}) \]

This statement is contradictory as compared to the initial assumption. Correspondingly we are able to confine this statement, since, there is no possibility of \( t^+ > t^\in \), therefore the overall tariff is additionally minimized. So that \( t^+ \) is the optimum selection.

**B:** In this scenario, we will prove that \( e^{\min} \) is also the optimal choice due to contradictions. We assume, there is a possibility of \( e^* > e^{\min} \) in this case the overall tariff is further decreased. It is represented by \( a^*_d(t) \) which is the optimum solution for \( e^* \). Therefore, we are again using tariff function \( G(a, \gamma^{\text{out}}) \) for the following case.

The implementation of \( e^* \) could lead towards new settlements referred as differentiation points, which are represented by \( t^{\in,\text{in}} \) and \( t^{\in,\text{out}} \) correspondingly. Two new water levels have been added at this stage, which are denoted by \( y^{\in,\text{in}} \) and \( y^{\in,\text{out}} \). In such scenario, EVs are supplying minimum power to the grid. It is possible due to the specialty of the water filling algorithm; it could be verified easily from following formulations:

\[ [t^{\in,\text{out}} < t^{\text{out}} < t^{\text{in}} < t^{\in,\text{in}}], \]

\[ [y^{\in,\text{out}} > y^{\text{out}} > y^{\text{in}} > y^{\in,\text{in}}] \]

In the following narration, these two novel water levels \( y^{\in,\text{in}} \) and \( y^{\in,\text{out}} \) equally shift away from the initial water level which is \( y^{\text{out}} \). By implementing the similar algorithm employed in scenario A, we get;

\[ \sum_{t=0}^{t^{\text{out}-1}} g_t(a^*(t), y^{\text{out}}) > \sum_{t=0}^{t^{\text{out}-1}} g_t(a^*, y^{\text{out}}) \]

\[ \sum_{t=t^{\in}}^{t^{\text{out}-1}} g_t(a^*(t), y^{\text{out}}) = \sum_{t=t^{\in}}^{t^{\text{out}-1}} g_t(a^*, y^{\text{out}}) \tag{27} \]

\[ \sum_{t=t^{\in}}^{t^{\in-1}} g_t(a^*(t), y^{\text{out}}) > \sum_{t=t^{\in}}^{t^{\in-1}} g_t(a^*, y^{\text{out}}) \]

By (28) we get:

\[ G(a^*, y^{\text{out}}) > G(a^*, y^{\text{out}}) \]

Even this possibility is also a contradictory statement which is inverse from our initial assumption. Thus, we can conclude the eventual result that \( e^* < e^{\min} \) does not exists so that the overall tariff is further reduced. Note that in this case \( e^{\min} \) is the optimal solution.

4. Simulation Results

In the following section we have briefly presented the simulation results, these simulations has been carried out to evaluate the performance of our proposed algorithm. Initially we have only penetrated five EVs to regulate the load demand of 100 household appliances, after that we have applied this algorithm by introducing 10 assorted EVs. In last, we have compared the results of our
proposed algorithm with the existing techniques to evaluate the robustness of the proposed algorithm.

4.1. Scenario 1: Penetrating 5 EVs

In this case, we have used 5 EVs to power 100 households. This basic power requirement and load demand are presented in Fig. 1. Let each EV coordinates with the grid at 20:00, and disconnects at 8 am in the next morning. The EVs specifications are as follows \( a_d^{-\text{in}} = 5\text{kW}, a_d^{-\text{out}} = 25\text{kWh}, G_d^{\text{max}} = 25\text{kWh}, \eta^{\text{in}} = 95\%, \epsilon = 25\%, \bar{e} = 90\%, \) and the output efficiency of EVs are \( \frac{1}{\eta^{\text{out}}} = 105\% \), whereas \( e_d^{-\text{in}} = 90\% \). We fix the sampling duration \( \Delta T = 20 \text{min} \). The initial SOCs of EVs are heterogeneous and denoted by \( e_1 = 33.1149\% \), \( e_2 = 20.7143\% \), \( e_3 = 36.9827\% \), \( e_4 = 38.6800\% \), \( e_5 = 33.5747\% \).

We fix \( e^{\text{min}} = \epsilon = 25\% \). The eventual gain of step 1 and step 2 is presented in Fig. 2. It could be analyzed that after executing two initial steps, the water level during EV discharging is high as compared to the fixed discharging model, i.e., \( \gamma^{\text{out}} > \gamma^{\text{in}} \). As it is also given in step 3, it is not required to select \( e^{\text{min}} \) again, because we can get the optimal load scheduling by easily performing the charging/discharging tasks, as presented in Fig. 3.

![Fig 2: The results of case 1 and case 2, while penetrating 5 EVs](image)

We have also compared the peak load diversion by introducing collective EVs charging/discharging, this process has been carried out using optimal valley filling technique to perform the charging task by penetrating Algorithm 1. It is easily verified from simulation analysis, despite of the “invariant water filling” from 20:00 to 22:00, when all EVs are charging, the response of our algorithm is identical since it is consistently maintaining water level, which is the optimal solution. This is also proved when results compared to the theoretical results of Algorithm 1, we can deduce that the load profile curve is much smoother when EVs was performing charging or discharging tasks.

4.2. Scenario 2: Penetrating 10 EVs

In this scenario, we have evaluated multiple EVs performance with the slightly higher penetration ratio. This is different from scenario 1, as 10 EVs has been penetrated in this particular test. The parameters of the proposed algorithm are given here, and the non-mentioned details are same, as used in scenario 1.

\[
ee(0)\%(\%) = 33.124 30.832 36.892 38.769 33.484 \\
35.245 34.753 27.755 33.290 23.533 \\
G^{\text{max}} (\text{kWh}) = 42.649 58.387 52.737 41.890 45.640 \\
50.927 59.250 59.387 43.253 59.312, \\
\eta^{\text{in}}\%(\%) = 95.620 90.240 93.900 86.670 89.750 95.250 92.820 95.660 92.320 85.290 \\
\frac{1}{\eta^{\text{out}} \text{(%})} = 112.75 108.66 110.03 115.67 107.83 111.88 116.77 109.87 108.55 108.66
\]

Initially we have selected \( e^{\text{min}} = \epsilon = 25\% \). The performance of scenario 1 and scenario 2 is presented in Fig. 4. It can be analyzed, that after executing two initial steps, the water level of the battery discharging is minimum when it is compared with the initial discharging process, i.e., \( \gamma^{\text{out}} < \gamma^{\text{in}} \). It shows that it is not essential for EVs to evaluate the optimal \( e^{\text{min}} \) by using third step which adopts the optimization of the bi-section. The eventual result is given in Fig. 5. it is evident that \( e^{\text{min}} \) is 22.04%.

It is also analyzed that our algorithm not only controls the homogeneous EVs, it also manages the heterogeneous once. This test show that, the overall regulated load demand profile curve is in a stable form, and it could be further analyzed that optimum load shifting while EVs charging/discharging is very effective as compared to the valley filling charging technique.
4.3. Scenario 3: Performance Evaluation of the Proposed Algorithm with Bonmin

In this scenario, we compared the results of our proposed algorithm with the existing integral programming algorithm. Because the constraints (6)-(10), could be categorized as MINLP constraint, therefore we have used the open source, non-linear mixed integral programming (BONMIN) which solves the general MINLP constraints very effectively, which has been actually used to compute the original constraint [37], [38]. Particularly, a hybrid approximation based algorithm (usually called B-Hyb) is selected for the implementation of BONMIN. In [39] a detailed description of this algorithm is presented.

The comparison of BONMIN executer and our proposed algorithm has been carried out by penetrating 1 to 10 homogenous EVs. We infer that each EV connects to the grid at 20:00, and leaves at 8 am in the next morning, (similar to the previous cases). The other simulated EVs parameters are same as scenario 1.

To eliminate the uncertainty factor, both algorithms are simulated in MATLAB, during testing, both algorithms converged successfully to the optimal solution; however their computational performances are totally different. The convergence time of the BONMIN and our proposed algorithm with different EV numbers is presented in Table 1.

Table 1: The Computational Time Comparison of our Algorithm with Bonmin

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Algorithm Time (sec)</th>
<th>Our Algorithm</th>
<th>BONMIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00899</td>
<td>39.71</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.01087</td>
<td>204.89</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.01125</td>
<td>159.94</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.01148</td>
<td>1652.58</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.01205</td>
<td>4288.35</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.01250</td>
<td>93495.85</td>
<td></td>
</tr>
</tbody>
</table>

It could be leniently analyzed that the computational speed of our proposed algorithm is significantly faster as compare to the BONMIN algorithm. Our algorithm takes only microseconds for computation, and this computational time do not exceed much, even when more EVs have been penetrated. Conversely, the computation time acquired by the BONMIN algorithm is significantly higher and gradually increases as more EVs penetrates. In order to govern the charging/discharging process of only 10 EVs it
5. Conclusion

In this research work, we have investigated the constraints restricting the broader deployment of EVs. Preliminarily we have formulated a load scheduling problem to manage the EVs charging/discharging, and this load scheduling problem has been defined as VDP constraint, which is extremely difficult to compute directly. Therefore, by altering the features of basic load profile of the night hours, we have formulated a computable estimation model for the VDP constraint, moreover, to leniently govern the computation of the following constraint; we have proposed a distributed algorithm to achieve optimal load scheduling for EVs. By implementing this algorithm, each EV is charged to the required SOC while keeping the overall power demand stable.

This algorithm is distributed in such manners that, primarily all EVs are locally managed before forwarding this information to the aggregator. In future, the electricity distributors would need to work for developing methodologies to maintain assumption 1, in those scenarios our pre-proposed load scheduling model can be employed. Otherwise numerous dynamic EVs networks would be required for this purpose. In addition to promote the use of EVs, a robust system is required where an EV should be allowed to easily connect and disconnect for charging. Moreover, in near future it is anticipated that existing conventional grids will also face extremely inconsistent energy generation penetration through intermittent renewable resources which is known as distributed generation even in this case our algorithm can be used to stabilize the grid by enabling the V2G facility.

6. References


