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Flexural Behaviour of Reinforced Fibrous Concrete Beams: Experiments and Analytical Modelling

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Abstract

Flexural behaviour of reinforced fibrous concrete beams was investigated in this research study. Two types of metallic fibers were studied: amorphous metallic fibers (FibraFlex fibers), and carbon steel hooked-end fibers (Dramix fibers). Four types of reinforced concretes were made: one control (without fibers) and three fibrous. Among three reinforced fibrous concretes, two contained fibers in mono form and one contained fibers in hybrid form. The total quantity of fibers in mono and hybrid forms was 20 kg/m^3 and 40 kg/m^3, respectively. Three point bending tests were performed according to European standards "NF EN 14651" on beams of 150 x 150 mm cross section and length of 550 mm. The results showed that due to positive synergetic interaction between the two metallic fibers used, reinforced fibrous concrete (RFC) beams containing fibers in hybrid form exhibited better response at all loading stages. Analytical model to predict ultimate moment capacity of the RFC beam of rectangular section was developed and is presented in this paper. Analytical results for ultimate moment were found to be in good agreement with experimental results.

Key Words: concrete; fibers; hybridization; ultimate moment capacity; analytical modelling

Notations

A_s	*area of steel reinforcing bars*
α_b	*bond efficiency factor*
f'_c	*compressive strength of concrete*
c	*depth of neutral axis*
a	*depth of compressive stress block*
d	*effective depth of beam specimen*
β	*factor defining the depth of the equivalent rectangular stress block for ordinary concrete*
γ	*factor defining the intensity of comp. stress of the equivalent rectangular stress block for ordinary concrete*
h	*height of Beam specimen*
z_1, z_2	*lever arm distances*
y'_c	*line of action of compressive forces*
MOR	*modulus of rupture*
α_o	*orientation factor*
T_s	*tensile force carried by steel bars*
T_f	*tensile force carried by fibers*
C	*total compressive force*
T	*total tensile force*
σ_f	*tensile strength of fibers*
M_u	*ultimate moment capacity*
σ_t	*ultimate tensile strength of fiber reinforced concrete*
ε_{cu}	*ultimate concrete strain in compression*
V_f	*volume fraction of fibers*
b	*width of beam specimen*
f_y	*yield strength of steel bars*
ε_y	*yield strain of steel*

1. Introduction

Concrete is characterized as a brittle material with low tensile strength and low strain capacity. Its mechanical behaviour is critically influenced by crack propagation. Problems related to concrete brittleness and poor resistance to cracking can be addressed by reinforcing plain concrete with randomly distributed fibers [1]. Many of the properties of fiber-reinforced concrete (FRC) can be used to advantage in the concrete flexural members reinforced with conventional bar reinforcement [2]. The use of steel fibers along with longitudinal steel bars improves the yielding moment, ultimate moment and post-yield behaviour. Moreover, addition of fibers reduces immediate deflection, long-term deflection and crack width of beam [3,4]. Many kinds of fibers have been used in concrete since last six decades and until now no single fiber reinforced concrete could exhibit perfect mechanical properties.

In recent years, attention has been given to hybridization of fibers in the concrete to get more enhanced response in terms of mechanical properties [5,6,7,8]. Hybridization means to mix two or more than two types of fibers in a matrix. The basic purpose of using fibers in hybrid form is to control cracks at different size level, in different zones of concrete and different loading stages [9]. The hybridization of fibers in concrete may be done by mixing fibers of different physical, geometrical and mechanical properties [8]. Most studied hybrid combinations of fibers include Steel-polypropylene, steel-carbon and carbon-polypropylene.

In the present experimental study, flexural behaviour of the RC beams containing two different metallic fibers in hybrid form has been investigated. Fibers used in this study differ in their geometrical, physical and mechanical characteristics.

Concrete reinforced with steel bars and randomly distributed fibers is named in this study as reinforced fibrous concrete (RFC). Basic purpose of this experimental research study is to obtain experimental data on the flexural strength of RFC beams containing metallic fibers in mono and hybrid forms at different loading stages. The experimental results are to be compared with analytical solution based on engineering practices in reinforced concrete calculation, in order to render fiber addition in concrete elements more attractive for practical applications.

2. Experimental Program

2.1 Materials

For all concrete mixes, CEM I 52.5 R type cement, course aggregates having size range of 4 to 10 mm, natural river sand with maximum particle size of 4 mm were used. To enhance fresh properties of FRC, a super-plasticizer (admixture) was used. The composition of the control concrete is given in Table 1.

Table 1: Mixture proportion of the control concrete

Material	Quantity, kg/m^3
Cement	322
Sand	872
Gravel	967
Water	193
Super-Plasticizer	1.61

2.1.1 Types of Fibers Used

FibraFlex Fibers: these fibers are produced by Saint-Gobain Seva, France and are made of amorphous metal. The fibers are shown in Fig.1. Due to their rough surface and large specific surface area, these fibers are characterized by a high degree of bonding with the concrete matrix.

Dramix fibers (Dramix ZP 305): these fibers are made using carbon steel wire and are characterized by weak bond with the matrix due to their smooth surface and less specific surface area compared to FibraFlex fibers. Dramix fibers are circular with hooked-ends as shown in Fig.2. The characteristics of fibers (FibraFlex and Dramix) are given in Table 2.

Fig.1: Amorphous metal fibers (FibraFlex)

Fig.2: Carbon steel hooked-ends fibers (Dramix)

Table 2: Fibers investigated in this study

Fiber	Fiber Type	Dimension (mm)				E, GPa	Tensile strength (MPa)	Cross Sectional shape
		L	W	T	D			
FibraFlex	amorphous metal	30	1.6	0.03	-	140	2000	Rectangular
Dramix	carbon steel	30	-	-	0.55	210	1200	Circular

Table 3: Fiber contents in different concrete mixtures

Beam	Mixture Type	Quantity of fibers, kg/m^3		Total quantity of fibers, kg/m^3	Compressive strength, MPa
		FibraFlex	Dramix		
RCB-Cont	Control	---	---	---	42
RCB-F20	mono fiber	20	---	20	44
RCB-D20		---	20	20	42
RCB-F20D20	Hybrid fiber	20	20	40	43

2.2 Concrete Composition

Four reinforced concretes: one control (without fibers) and three containing fibers were made and tested. Among the three reinforced fibrous concretes, two contained single fiber and one contained fibers in hybrid form. Quantity and type of fiber for each concrete mix are given in Table 3. Compressive strength value at the age of 28 days of all concrete mixtures is also given in Table 3. For the nomenclature of different concrete beams given in Table 3, RCB stands for Reinforced Concrete Beam, Cont stands for control concrete, F stands for FibraFlex fibers, D stands for Dramix fibers and 20 or 40 is the dosage of fibers in kg/m^3.

2.3 Test Specimens

A total of 8 notched beams (2 beams for each concrete mix) with cross section of 150 x 150 mm and total length of 550 mm were constructed. Each beam was reinforced with two 6 mm diameter steel bars (fy = 500 MPa) fulfilling the minimum requirement of Eurocode 2 for the tension steel. The cross section and reinforcement details of tested beam specimen are shown in Fig.3.

2.4 Testing Procedure

28 days after the casting, three point bending tests were performed on beams according to European Standards [10]. Although this standard is for fibered concrete without steel bars, the same procedure has been adopted here for RFC beams. All the tests were controlled by crack mouth opening displacement (CMOD) using LVDT. In each test, mid-span deflection of the beam was also measured using LVDT. CMOD and deflection measurement scheme is shown in Fig.4.

3. Test Results

Experimental results (all compositions) about cracking and ultimate moment (moment corresponding to peak load attained by each composite) are given in Table 4 and moment- CMOD and moment- deflection curves for each specimen are shown in Fig.5 and Fig.6, respectively. It can be observed that for a given CMOD or deflection value, the load carrying capacity of the reinforced concrete beam with metallic fibers is significantly improved.

Fig.7 shows the values of cracking and ultimate moment for all concretes while strength effectiveness with the addition of metallic fibers is shown in Fig.8. Compared to control beam (RFC-Cont), the cracking moment is increased by 36.8% and 0.3% for RCB-F20 and RCB-D20 beams, respectively. This shows that cracking moment is increased significantly in the presence of FibraFlex fibers. On the contrary, Dramix fibers do not affect the cracking moment.

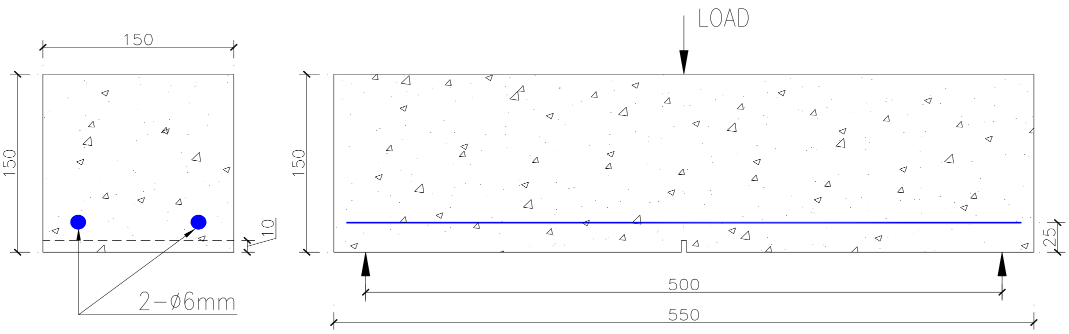


Fig.3: Cross section and reinforcement details of test specimen

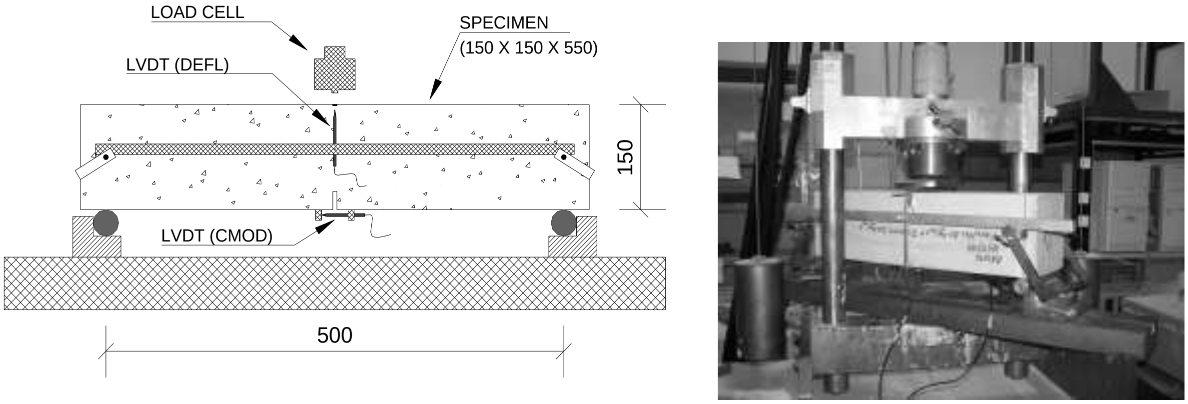


Fig.4: Testing setup for three point bending tests

Table 4: Cracking and Ultimate Moments for all compositions

Beam Type	Sample #	Cracking Moment, M_{cr} (kN-mm)	Ultimate Moment* M_{ult} (kN-mm)
RCB-Cont	1	2379	5831
	2	2281	5669
RCB-F20	1	3216	5874
	2	3163	6113
RCB-D20	1	2333	6138
	2	2338	6228
RCB-F20D20	1	3653	7029
	2	3292	6974

** Maximum value in CMOD-Moment or Deflection-Moment curve*

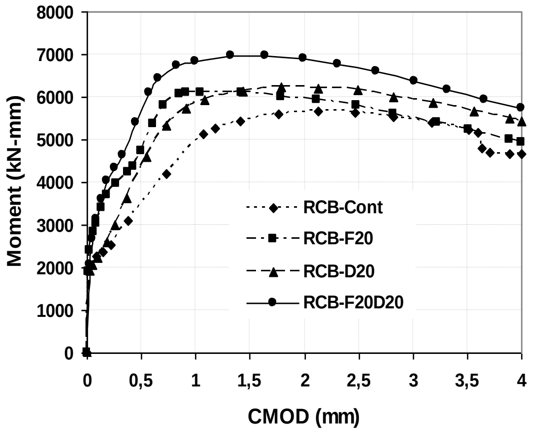


Fig. 5: Moment versus CMOD curves

Fig.8 also shows that the ultimate moment of reinforced concrete increases with the addition of metallic fibers. Compared to RCB-Cont, the ultimate moment is increased by 4.1% and 8.5% for RCB-F20 and RCB-D20 beams, respectively.

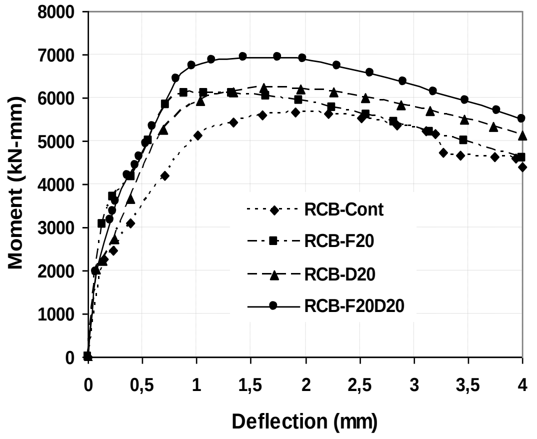


Fig.6: Moment versus Deflection curves

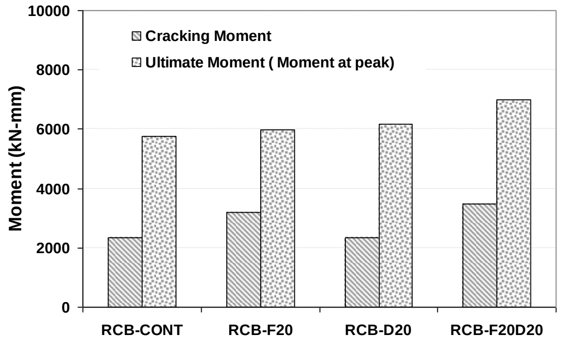


Fig. 7: Cracking and ultimate moment (experimental results)

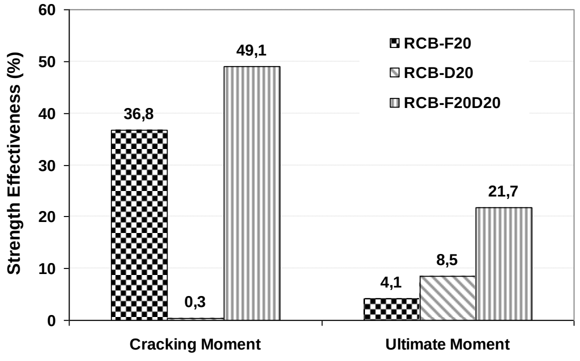


Fig.8: Effectiveness of fibers (Cracking and ultimate moment)

The variation of CMOD and deflection with the increase of moment is shown in Fig.9 and Fig.10, respectively. It can be seen that for a given moment, the addition of metallic fibers significantly reduced the crack opening and deflection. The deflection value at ultimate moment was 2.8 times less in RCB-F20 compared to RCB-Cont. Similarly, it was 2.2 times less in RCB-D20 compared to RCB-Cont. Similar observations were also made for crack opening. CMOD value at ultimate moment was 2.6 times less in RCB-F20 and 2.1 times less in RCB-D20 compared to RCB-Cont.

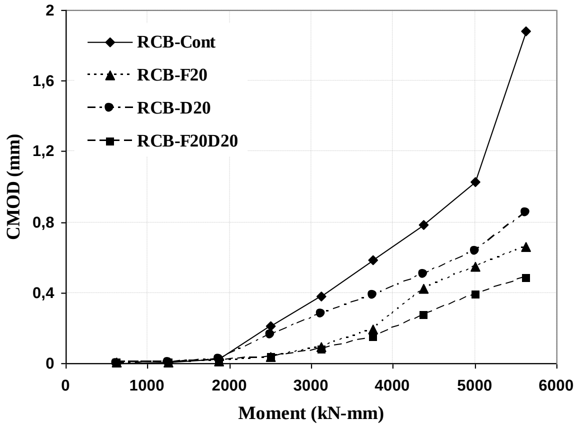


Fig.9: Variation of CMOD with moment (up to M_{ult} attained by RCB-Cont)

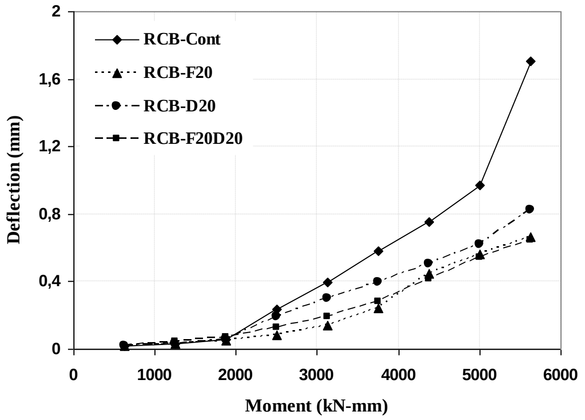


Fig.10: Variation of deflection with moment (up to M_{ult} attained by RCB-Cont)

4. Flexural Capacity of Beams: Analytical Approach

Simple mechanics and equilibrium conditions have been used to calculate the flexural capacity in terms of cracking and ultimate moment of the beams containing randomly distributed metallic fibers and reinforcing steel bars. The tensile forces carried by the added fibers create an additional internal moment capacity which is simply added to the moment capacity of reinforced concrete beam section. The analysis is based on following main assumptions: (1) Plane section remains plane after bending; (2) the tensile forces balance the compressive forces, (3) the internal moment is equal to external applied bending moment; (4) perfect bond between reinforcement and the surrounding concrete.

4.1 Cracking Moment

Simplified procedure proposed by Campione [11] is used to calculate the cracking moment. Neglecting the effect of steel bar, the cracking moment is calculated using Eq.1.

$$M_{cr} = \frac{bh^2 \times MOR}{6} \qquad (1)$$

Where, MOR is modulus of rupture of fiber reinforced concrete obtained experimentally. Additionally, a separate series of 3 point bending tests on prisms of cross section 100 x 100 mm and length of 550 mm (span = 500 mm) was also carried out in order to determine the modulus of rupture of concrete compositions used in this study. For each composition, three samples were tested. These tests were performed according to European standard [10] with the exception of specimen size: the standard cross section of test specimen is 150 x 150 mm whereas for MOR test in the present study, it was 100 x 100 mm. The results (average of three samples) are shown in Fig.11.

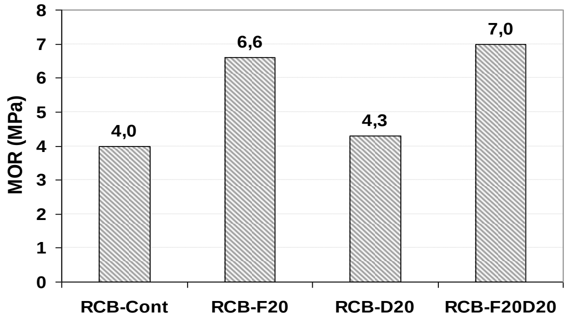


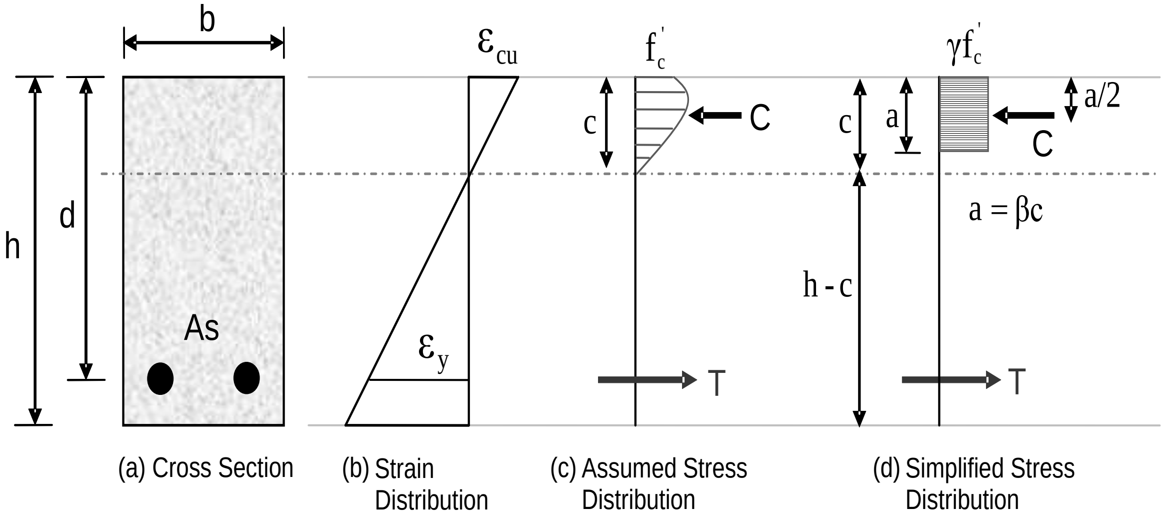
Fig.11: Modulus of Rupture (MOR)

4.2 Ultimate Moment

The assumed and simplified stress distribution of reinforced non-fibrous and fibrous concretes at failure is shown in Fig.12 and Fig.13 [2, 12]. In RFC beam, steel reinforcing bar, concrete matrix and randomly distributed fibers contribute to carry the post-cracking tension. Referring to the assumed and simplified stress distribution in RFC beam rectangular section shown in Fig.13 where parabolic compressive stress zone is divided into two parts: rectangular and triangular, the ultimate moment is calculated using Eq.2.

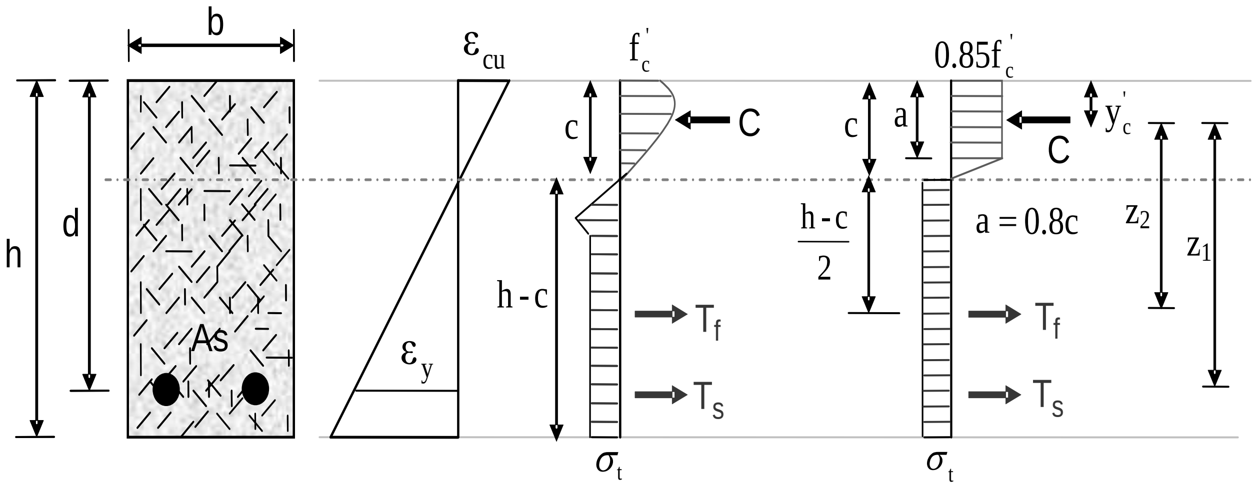
$$M_u = T_s \times z_1 + T_f \times z_2 \qquad (2)$$

In Eq.2, T_s and T_f are tensile forces carried by steel bar and fibers, respectively and z_1 and z_2 are the respective lever arm distances. For RFC beam



(a) Cross Section (b) Strain Distribution (c) Assumed Stress Distribution (d) Simplified Stress Distribution

Fig.12: Stress strain distribution (RC section)



(a) Cross Section (b) Strain Distribution (c) Assumed Stress Distribution (d) Simplified Stress Distribution

Fig.13: Stress strain distribution (RFC section)

containing fibers in hybrid form, the ultimate moment capacity is calculated by simply adding the contribution of individual fibers in carrying tension and the expression is given in Eq.3

$$M_u = T_s \times z_1 + (T_{f1} + T_{f2}) \times z_2 \qquad (3)$$

T_{f1} and T_{f2} are the tension carried by each fiber used in hybrid combination. T_s and T_f are calculated using Eqs.4 and 5.

$$T_s = A_s \times f_y \qquad (4)$$

$$T_{f_{1,2}} = \sigma_{t1,2} \times b \times (h - c) \qquad (5)$$

In Eq.5, σ_t is ultimate tensile strength of fiber reinforced concrete and is greatly influenced by the properties and contents of fibers.

4.2.1 Determination of z1 and z2

In the following section, step by step procedure is given to calculate z_1 and z_2. In the following equations, C_1 is the compressive force corresponding to the area of the rectangular compressive stress block and C_2 is the compressive force corresponding to the area of triangular compressive stress block as shown in Fig.13.

$$C_1 = 0.85 f_c^{'} \times 0.80cb = 0.68 f_c^{'} cb \qquad (6)$$

$$y_1 = (0.80 \times c)/2 = 0.4c \qquad (7)$$

$$C_2 = \frac{1}{2}(0.85 f_c^{'} \times 0.20c)b = 0.085 f_c^{'} cb \qquad (8)$$

$$y_2 = \frac{1}{3}(0.20 \times c) + 0.80c = 0.867c \qquad (9)$$

$$\sum C = C_1 + C_2 = 0.765 f_c^{'} cb \qquad (10)$$

$$y_c^{'} = \frac{C_1 y_1 + C_2 y_2}{\sum C} = 0.452c \qquad (11)$$

$$z_1 = d - y_c^{'} \qquad (12)$$

$$z_2 = (\frac{h - c}{2}) + (c - y_c^{'}) \qquad (13)$$

Depth of the neutral axis "c" is determined using Eq.14 for RCB beam containing fibers in mono form and using Eq.15 for RCB beam containing fibers in hybrid form. Eqs.14 and 15 are obtained by equating compression and tension forces i.e., $C = T_s + T_f$.

$$c = \frac{\sigma_t bh + A_s f_y}{0.765 f_c^{'} b + \sigma_t b} \qquad (14)$$

$$c = \frac{(\sigma_{t1} + \sigma_{t2})bh + A_s f_y}{0.765 f_c^{'} b + (\sigma_{t1} + \sigma_{t2})b} \qquad (15)$$

4.2.2 Determination of σ_t

The ultimate behaviour of FRC is governed by properties of fibers, number of fibers, etc. According to Hsu et al [13], number of fibers acting across the cross section is determined by the following expression:

$$N = \alpha_o \times \frac{V_f}{A_f} \qquad (16)$$

From the Eq.16, NA_f is the area of fibers per unit area of section and is equal to $\alpha_o V_f$. A_f is area of fibers and V_f is volume fraction of fibers. In the analysis of reinforced fibrous concrete, each fiber is considered as small longitudinal reinforcement present through the whole length of section.

The ultimate tensile strength of fiber reinforced concrete is calculated by using the following equation:

$$\sigma_t = \alpha_o \times V_f \times \sigma_f \times \alpha_b \qquad (17)$$

Where, α_o is orientation factor and is equal to 0.41 [14], α_b is bond efficiency factor and its value varies from 1 to 1.2 depending upon fiber characteristics [15]. For straight fibers, the value of α_b is taken equal to 1 [16], but in this study, for FibraFlex fibers which are straight, taking into account high bond strength with matrix due to their surface roughness, the value of α_b is taken equal to 1.2; the maximum value proposed by ACI Committee. For hooked-ends fibers (Dramix), Dancygier et al. [17] proposed the value of α_b equal to 1.2. In this study, the same value (i.e., α_b =1.2) is used. σ_f is tensile strength of fibers and the values for the fibers used in this study are given in Table 2. Analytical values of ultimate moment capacity are given in Table 5 along with ratio between experimental and analytical values. It is pertinent to mention that the ultimate moment capacity of hybrid fiber-reinforced concrete specimen calculated using proposed equation is almost similar to experimentally obtained moment capacity.

Table 5: Analytical and experimental values of ultimate moment capacity

Composition	Ultimate Moment (kN-mm)		$\frac{(M_{ult})_{anl}}{(M_{ult})_{exp}}$
	$(M_{ult})_{exp}$	$(M_{ult})_{anl}$	
RCB-F20	5988*	5225	0.872
RCB-D20	6238*	5101	0.817
RCB-F20D20	7000*	6861	0.980

** Value is average of two samples*

5. Discussion

The experimental results show that the ultimate moment resistance of reinforced concrete beam is increased by 4.0 to 7.5 % with the addition of randomly distributed FibraFlex fibers. Fiber action to stop propagation of cracks at micro or macro level strongly depends on the properties of fiber (geometry, strength and stiffness) and on the bond between fibers and concrete matrix. For the bond between matrix and fiber, matrix compactness also plays an important role. Among the two fibers used in this study; FibraFlex fibers develop good bond with the matrix because of their rough surface and large specific surface area. According to a study carried out by Turatsinze et al [18], micro-cracking occurs inside the matrix before the peak load in flexure, in this context, FibraFlex fibers act as soon as the first micro-cracks open and immediately restrain their propagation due to good bond with concrete matrix. By this way, they enhance the response prior and just after the peak load. With further increase of crack opening, the fibers contribute to carry tension along with steel bar and response of the composite is improved in term of load bearing capacity, reduced deflection and smaller crack opening (Fig.9 and Fig.10). With regard to the failure of the FibraFlex fibers is concerned, when the stress in the fiber exceeds its tensile strength with the increase of crack opening, the fibers break instead of pulling out from the matrix and the post-peak residual load bearing capacity approaches to a value equal to RC beam without fibers (Fig.5).

Since the tests were performed on notched beams, during each test a single tensile crack from the notch area was observed to initiate and propagate in upward direction towards compression zone as shown in Fig.14.

Dramix fibers develop poor bond with concrete matrix due to their smooth surface and small specific surface area [19]. As a result of poor bond, at micro-cracking, these fibers slip from the concrete matrix and minor positive effect on the response of the composite in terms of strength is observed. Since these fibers are hooked-end, sufficient anchorage in the matrix helps these fibers to restrain macro-cracks propagation by transferring the stress across the crack; as a result, post-crack response (toughening effect) of RCB beam is significantly improved. With the increase of crack opening, these fibers are stretched and anchorage in the concrete matrix due to hooked-end is further improved and fibers play important role in carrying tension along with steel reinforcing bar at macro-cracking stage. After the yielding of steel bars, crack opening is increased significantly and fibers are pulled out (slipped) from the concrete matrix instead of breaking and their hooked-ends turn straight.

Since the two metallic fibers used in this study provide reinforcement at different levels [19]: FibraFlex fibers at micro-cracking level and Dramix fibers at macro-cracking level, RFC beams containing both fibers in hybrid form exhibit maximum improvement in flexural strength and toughness because of positive synergetic interaction between fibers.

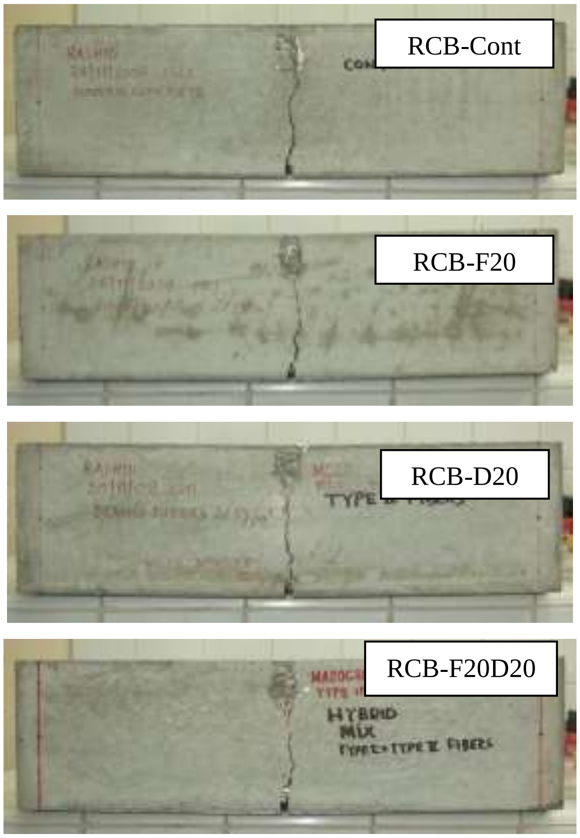


Fig.14: Cracking of RC beams with and without fibers

Analytical model developed in this study to predict ultimate moment capacity of RFC beam is a simple and more practical approach and thus a useful tool for engineers to assess the bending capacity of RC beams containing fibers. Results in terms of cracking and ultimate moment obtained using presented analytical model show good agreement with experimental results. The ratios between analytical and experimental values of ultimate moment lie between 0.8 & 1.0 for different reinforced concrete beams tested in this study.

6. Conclusions

Based on the test results and predicted values of cracking and ultimate moment capacities, the following conclusions are drawn:

- The level of improvement in the flexural response of the RFC beam greatly depends on physical and mechanical properties of metallic fibers.
- A significant reduction in the crack width and deflection of RC beam is guaranteed by the addition of metallic fibers.
- The matrix-fiber high bond strength of amorphous metallic fibers (FibraFlex) makes the fibers more effective in strengthening the composite. In opposite to that ductile behaviour and hooked-ends of carbon steel fibers (Dramix) make the fibers more effective in toughening the composite.
- Positive synergetic effect exists between the two metallic fibers used in this study. Therefore, composite containing these fibers in a well chosen hybrid form can exhibit high performance in terms of strength and toughness.
- Analytical model developed to predict flexural strength of RFC beam with rectangular cross section containing fibers in hybrid form is a simple and practical tool for the engineers. However, this needs more validation by comparing results from other researchers.

7. Acknowledgements

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